33.51 **A $4in$ aluminum sphere is dropped into a deep body of water. The coefficient of drag is 0.5. What is the terminal velocity of the sphere?**

A. $4\frac{ft}{s}$

B. $7\frac{ft}{s}$

C. $8\frac{ft}{s}$

D. $15\frac{ft}{s}$

Draw a free body diagram for the sphere. There are 3 forces in play: (1) the downward force of gravity, (2) the upward force of buoyancy, and (3) the upward force of drag. At terminal velocity, there is no acceleration, therefore the 3 forces are in balance.

$$F_g = F_B + F_D$$

Start by finding the force of gravity on the sphere. The density of aluminum can be found in the Properties of Metals table. Recall or look up the formula for volume of a Sphere.

$$\rho_{aluminum} = 168\frac{lb_m}{ft^3}$$

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{2}{12}ft\right)^3 = 0.0194ft^3$$

$$F_g = \frac{mg}{g_c} = \frac{\rho V g}{g_c} = \frac{\left(168\frac{lb_m}{ft^3}\right)\left(0.0194ft^3\right)\left(32.2\frac{ft}{s^2}\right)}{32.2\frac{lb_m \cdot ft}{lb_f \cdot s^2}} = 3.26lb_f \downarrow$$

Next find the force of Buoyancy, which is the product of the volume of fluid displaced by the sphere and the specific weight of the fluid, in this case water. Note that since the specific weight of aluminum is greater than water, the sphere sinks, and 100% of its volume displaces water. An object that floats on the surface displaces only a percentage of its total volume, enough to equal the object's weight.

$$F_b = \gamma V_{displaced}$$

$$F_B = \left(62.4\frac{lb_f}{ft^3}\right)\left(0.0194ft^3\right) = 1.21lb_f \uparrow$$

Subtract to find the Drag Force.

$$F_D = F_g - F_B = 3.26lb_f - 1.21lb_f = 2.05lb_f \uparrow$$

Rearrange the equation for drag to solve for velocity. Calculate the frontal area for the sphere, which is the area of a circle of the same diameter. Solve for the terminal velocity.

$$F_D = \frac{C_D \rho v^2 A}{2 g_c}$$

$$v = \sqrt{\frac{F_D \left(2\right) g_c}{C_D A \rho}}$$

$$A = \frac{\pi}{4} D^2 = \frac{\pi}{4} \left(\frac{4}{12} ft\right)^2 = 0.087 ft^2$$

$$v = \sqrt{\frac{\left(2.05 lb_f\right) \left(2\right) \left(32.2 \frac{lb_m \cdot ft}{lb_f \cdot s^2}\right)}{\left(0.5\right) \left(0.087 ft^2\right) \left(62.4 \frac{lb_m}{ft^3}\right)}} = 6.97 \frac{ft}{s}$$

Answer B